Resonant Load Control Methods for Industrial Servo Drives

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ABSTRACT - High-performance servo drives are often limited by mechanical load resonance. In this paper, seven methods of resonant-load control are compared for their ability to improve performance in the presence of a low-frequency (100 Hz) lightly damped resonance. Several of the methods are based on filtering the command signal; the remaining methods are based on signals acquired from an observer. Development of these methods is presented; each method is applied to a physical system and evaluated for effects on command response and dynamic stiffness.

I. INTRODUCTION
Servo drives are used in a wide range of industrial applications including metal cutting, packaging, textiles, web-handling, automated assembly and printing. Command response and dynamic stiffness are two key performance ratings for high-performance applications. Designers use closed loop controllers such as proportional-integral, PI, velocity loops in servo systems. Such controllers must be configured with high gains for the system to achieve high performance.

Mechanical resonance is one of the most common problems designers face when trying to maximize either command response or dynamic stiffness [1]. Mechanical resonance is usually caused by a combination of high servo gains and a compliant coupling between the motor and load. The compliant coupling comes from the transmission which is some combination of components such as mechanical couplings, shafts, gearboxes, lead screws, and belt/pulley sets. The mechanical stiffness of these components is limited; if the inertia of the transmission components is small compared to the motor and load, the stiffness of the components can be treated as a single, composite, equivalent spring constant that interconnects motor and load, as shown in Fig. 1.

Fig. 1. Motor and load with a compliant coupling

A block diagram of the compliantly coupled mechanism is shown in Fig. 2. Here, the equivalent spring constant of the transmission, $K_S$, is shown as providing torque to the load in proportion to the difference of motor and load positions. Also, to represent loss producing properties, a mechanical damping term is shown producing torque in proportion to velocity differences via cross-coupled viscous damping, $b_s$.

Fig. 2. Block diagram of compliantly coupled load

The transfer function from electromechanical torque, $T_E$, to motor velocity, $V_M$, is

$$\frac{V_M}{T_E} = \frac{1}{J_M + J_L} s^2 + \frac{b_s s + K_S}{J_L J_M s^2 + b_s s + K_S}$$

which can be viewed as a pure, lumped inertia, $1/(J_M+J_L)s$, term on the left and a dual quadratic or “bi-quad” function on the right. This is the transfer function that would represent the plant in the case where the sole position feedback sensor was on the motor (as oppose to the load), as is common in industry.

PI and PID controllers are designed to control inertial terms. The bi-quad function causes instability by altering the phase and gain of the lumped inertial plant. The viscous damping, $b_s$, on most practical machines is low so that both the numerator and denominator are lightly damped. This produces a very low gain at the anti-resonant frequency, $\omega_{AR}$, the frequency for which the numerator is minimized. It also produces a very high gain at the resonant frequency, $\omega_R$, the frequency for which the denominator is minimized. The undamped values of these frequencies are shown in (2).

$$\omega_{AR} = \sqrt{\frac{K_S}{J_L}} \text{ rad/s} \quad \omega_{R} = \sqrt{\frac{K_S}{J_L J_M + J_L + J_M}} \text{ rad/s}$$

The effect of the bi-quad term can be seen in Fig. 3, where, without it, the gain would be constantly declining at
20 dB/decade (shown as dashed line) and the phase would have a fixed 90° lag, characteristic of inertial loads. Methods of controlling resonance rely on modifying the effects of the bi-quad term.

Methods of controlling resonance rely on modifying the effects of the bi-quad term.

A common control structure for servo systems is to use a PI velocity controller, cascaded with anti-resonance filters, to drive a very high bandwidth current regulator. The most common anti-resonance filters in are based on one of three filtering techniques: low pass, notch, and bi-quad filters. The high bandwidth current regulator produces electromagnetic torque to drive the motor/load mechanism. The motor position is read from an encoder or other motor position sensor, and used to calculate a sample average velocity feedback signal for the control loop. Such a system is shown in Fig. 4.

Figure 4 shows two observers, both fed by the encoder position and electromagnetic torque. One is a rigid-body observer which observes the motor velocity, $\dot{V}_M$, and acceleration, $\ddot{A}_M$, assuming no knowledge of the load inertia or coupling stiffness. The second is a compliant-body observer which models the motor and load velocities, $\dot{V}_M$ and $\dot{V}_L$, respectively, and accelerations based on knowledge of the load and coupling characteristics.

Many methods have been used to reduce the effects of resonance [3]. This paper will compare seven methods, i.e., three filtering and four observer-based methods, for their ability to control a lightly damped, low-frequency (100 Hz) resonance. The methods discussed here will be limited to using a single motion sensor—a position sensor on a motor.

The evaluation will be based on the ability to provide increased dynamic stiffness and faster command response. In addition, each method will be evaluated for sensitivity to mechanical changes. The evaluation is done using a combination of an experimental system and a simulation model.

II. FILTERING METHODS

Low-pass filter.

The low-pass filter is the most common method used to control resonance today. The low-pass filter is used to reduce the gain at the resonant frequency as shown in Fig. 5. This improves the gain margin at or near the resonant frequency; it also degrades the phase margin since the filter will reduce phase where the gain crosses over (about 30 Hz in Fig. 5).

The key advantage of the low-pass filter is that it is easy to use. Only the break frequency needs to be adjusted for the given load resonance. Unfortunately, the low-pass filter is not very effective for the commonly found case of low-frequency resonances, where the resonant frequency is no higher than 5 or 10 times the velocity loop bandwidth.

Notch filter.

Another widely used filter is the notch filter [4]. It has the transfer function:

$$T_N(s) = \frac{s^2 + \omega_N^2}{s^2 + 2\zeta\omega_N s + \omega_N^2}$$

(3)

Usually $\omega_N$ will be selected to approximate the load resonant frequency, $\omega_R$, and the damping ratio, $\zeta$, will be moderate to low, typically below 0.4. The notch filter, like the low-pass, is used to increase gain margin by attenuating the open-loop gain in the frequency region near the resonant frequency. The damping ratio is generally selected such that less phase distortion occurs at lower frequencies than when using a low pass. This allows higher gains in the control loop. The notch filter also passes the frequencies above the resonant frequency. The effect of the notch filter on the open loop frequency response is shown in Fig. 6.
Bi-quad filter.

The bi-quad filter is designed to cancel the effects of the physical bi-quad term of (1). It has the form

$$ T_{BQ}(s) \equiv \frac{s^2 + 2\zeta_N \omega_N + \omega_N^2}{s^2 + 2\zeta_D \omega_D + \omega_D^2} \quad (4) $$

The following equations are used to achieve cancellation:

$$ J_P = \frac{\hat{J}_L \hat{J}_M}{(\hat{J}_L + \hat{J}_M)} \quad \text{where the circumflex, \(\hat{\cdot}\), denotes an estimated parameter} $$

$$ \omega_N = \sqrt{\frac{K_D}{J_P}} \quad \omega_D = \sqrt{\frac{K_D}{J_L}} \quad \zeta_N = \frac{b}{(2 J_P \omega_D)} \quad \zeta_D = \frac{b}{(2 J_L \omega_D)} $$

If complete cancellation is achieved, the effect of the bi-quad filter is to eliminate the bi-quad term from (1), leaving (1) as an ideal inertial load. The command response and dynamic stiffness can be enhanced considerably. Theoretically, the bi-quad filter offers designers the greatest opportunity to expand command response and dynamic stiffness of the filtering methods.

The bi-quad filter has two major shortcomings. First, while the motor position/velocity may be controlled without oscillation, the load position, which is connected to the motor through a resonant coupling, still resonates as shown in Fig. 7.

The second shortcoming of the bi-quad filter is that the servo system is very sensitive to parameter changes. If mechanical parameters such as load inertia or spring constant change, the servo loop may become unstable.

III. OBSERVERS AS SENSOR REPLACEMENTS

The remaining anti-resonance methods require one or more signals in addition to the motor-mounted encoder. Most applications cannot afford additional sensors because of the cost of purchasing and mounting sensors, and the loss of reliability from extra components and wiring. This section will discuss two observers that can be used to obtain signals in addition to the encoder. The following section will discuss how those signals can be employed to reduce resonance.

Rigid-body Luenberger observer

The rigid-body Luenberger observer can be used to provide observed motor velocity, \(\hat{V}_M\), and acceleration, \(\hat{A}_M\). The Luenberger observer, as shown in Fig. 8, has a structure directly analogous to the motor. The electromagnetic torque, \(T_E\), as commanded by the drive acts as a feedforward input to the observer. It is divided by the estimated motor inertia to produce observed acceleration; that signal is integrated to produce \(\hat{V}_M\) and then observed position, \(\hat{P}_M\). Thus, the observer’s feedforward path is acting in parallel to the actual motor. Observer state feedback is used to force it to track the measured position. The difference between \(P_M\) and the motor position, \(\hat{P}_M\), is the observer error. This error term is compensated by a PID control law, in an attempt to drive that error to zero. When the error is sufficiently small, \(\hat{P}_M\) will be representative of \(P_M\) as well as \(\hat{V}_M\) and \(\hat{A}_M\) of the motor velocity and acceleration.

$$ \hat{P}_M = \frac{T_E}{J_M s} + \frac{1}{J_M} \hat{A}_M + \frac{1}{s} \hat{V}_M $$

The Eigenvalues of the observer can be set as equivalent to a three-pole Butterworth (maximally flat) filter by setting:

$$ K_{OD} = 2 \omega_N \quad K_{OP} = \omega_N \quad K_{OI} = \omega_N^2/2 \quad (6) $$

By rewriting the block diagram of Fig. 8, it can be seen that

$$ \hat{V}_M = \frac{T_E}{J_M s^3} \times \frac{s^3}{s^3 + K_{OD}(s^2 + K_{OP} s + K_{OI})} + \frac{1}{s} \hat{A}_M $$

$$ P_M s^3 \times \frac{K_{OD}(s^2 + K_{OP} s + K_{OI})}{s^3 + K_{OD}(s^2 + K_{OP} s + K_{OI})} $$

which upon inspection is a high-pass filtered velocity signal from the torque and a low-pass filtered velocity signal from the encoder. Similar to the low-pass filtered method of Section II, the observer removes high frequency content of the
encoder signal to reduce resonance problems. Unlike the filter, the observer fills in missing frequency content with information from $T_E$ which enables it to produce zero phase lag at lower frequencies and higher frequencies.

An alternative form of the Luenberger observer, the “extended Luenberger observer” is shown in Fig. 9 [5, 6, 7]. This structure is built recognizing that the process of differentiation in the PID controller can be avoided by feeding the derivative signal before the second integration. This reduces computational resources and reduces quantization noise while providing the equivalent term $V_M^*$. The acceleration term is effectively filtered since $K_{OD}$, the high-frequency term of the PID compensator, is removed from the forward path from $P_M$ and $P_T$ to $A_M$.

![Fig. 9. Extended rigid-body Luenberger observer](image)

**Compliant-body observer**

An alternative to the rigid-body observer is to use a model that includes the motor, load, and the equivalent, compliant, visco-elastic coupling. Here, the resonant model from Fig. 2 is coded within the observer, as shown in Fig. 10.

![Fig. 10. Compliant-body observer](image)

The principles of the compliant-body observer are similar to those of the rigid-body observer. This observer also can observe motor velocity and acceleration ($\hat{V}_M$ and $\hat{A}_M$) and load velocity ($\hat{V}_L$).

**Observer error**

An observer can be evaluated by how well it predicts the motor velocity in the presence of a torque disturbance. For example, the error of observer velocity, $V_M - \hat{V}_M$, as a function of torque disturbance is shown in Fig. 11 for the rigid-body observer. The error will be low at low frequency because of the integral term ($K_{OI}$) and at high frequency because the inertia reduces the effects of the disturbance at low frequency. The error will be largest in the midrange frequencies.

![Fig. 11. Frequency response of observed velocity error vs. disturbance torque](image)

**IV. Observer-Based, Anti-Resonance Methods**

This section will discuss anti-resonance methods based on using one or more observed signals in addition to the motor feedback device. Note that a related method uses observed torque disturbances to reduce vibration [8,9].

**Acceleration feedback.**

Acceleration feedback effectively increases the motor inertia [5, 6, 7, 10, 11]. Its conceptual implementation for the resonant load is shown in Fig. 12. By combining the forward and feedback paths, it can be seen that the effective motor inertia increases to $(1 + K_A) \times J_M$. Increasing physical inertia is a well-known method of reducing sensitivity to mechanical resonance. Acceleration feedback produces similar benefits without the negatives of physical motor mass such as increased weight and size and reduced peak acceleration.

![Fig. 12. Acceleration feedback](image)

Acceleration feedback is implemented by adding to the current command a term proportional to observed acceleration:

$$I_C := K_A \times \hat{A}_M / K_T$$  \hspace{1cm} (8)

where $I_C$ is the current command and $K_T$ is the motor torque constant. Quantization noise in the observed acceleration and phase lag in the current regulator limit how much acceleration feedback is practical. In the experiments run for this paper, values of $K_A > 2.5$ caused instability.

An alternative to using an observed acceleration signal is to measure average acceleration by double differentiating position. This is inferior to the observer because differentiation maximizes quantization noise and induces phase lag. In
these experiments, the observed acceleration signal allowed about twice as much acceleration feedback (about double $K_A$) as was allowed using average acceleration.

Another way of using acceleration feedback is to reduce motor inertia to zero by setting $K_A = -1$ [12]. This reduces $J_\text{P}$ in Fig. 12 to zero and thus eliminates the oscillatory behavior. This method has been observed to lack robust operation, for example, when the load experiences stiction [13]. Here, the apparent load inertia becomes infinite, and the PI controller, having had the motor inertia removed, is driving a spring; such a structure is unstable. Also, the method does not work well when there is significant phase lag in the acceleration signal. Neither the model nor the physical test setup were able to function with $K_A = -1$ using observed or average acceleration signals.

Observer filtering
Observer filtering here refers to the use of the observer to filter high-frequency signals from the encoder, filling in the missing information with the electromagnetic torque signal (7). This is superior to ordinary low-pass filters since it should yield zero phase lag in the observed velocity at lower frequencies and thus has less effect on loop stability.

Active resonance damping
Active resonance damping adds a torque in proportion to the difference of motor and load observed speeds

$$T_C = (\hat{V}_M - \hat{V}_L) b_{ADO} \quad (9)$$

Active damping increases the effective physical damping, $b_s$, similar to the way acceleration feedback increases effective inertia [14]. Active damping is well known to cure resonance when a physical sensor is placed on the load. The question addressed here is whether the compliant-body observer can be used to provide load and motor velocity allowing use of active damping without an additional sensor.

Center-of-mass control
Center of mass control uses the compliant-body observer to provide motor and load velocity. The velocity of the center of mass is then

$$V_{COM} = \hat{J}_M \times \hat{V}_M + \hat{J}_L \times \hat{V}_L \quad (10)$$

Using $V_{COM}$ reduces sensitivity to resonance between motor and load because during such resonance, the center-of-mass does not move. Center-of-mass control is commonly employed with separate sensors. The difficulty in using COM control is that, with a single actuator, it is destabilizing. In fact, (10) is similar to negative (destabilizing) active damping (9).

V. EXPERIMENTAL RESULTS

Test system
In order to test the resonant load control alternatives discussed in this paper, a physical system was constructed using a motor driving a load through a compliant coupling. The controller was a combination of a PC connected to a Kollmorgen ServoStar CE-03 amplifier configured as a torque (current) drive. The interface card was a Servo-to-go Model 2. The control algorithms were coded in the C programming language using floating point math. They executed every 250 uSec. A single program ran both observers simultaneously and all methods could be run in parallel.

The motor was selected as a Kollmorgen MT304A with a $K_T = 0.985 \text{ Nm/A-rms}$. The feedback device was a sine-encoder configured for a resolution of $10^6$ counts/rev. The compliant coupling was a steel rod 14mm in diameter and 85 cm long (about 80 cm exposed between inertial loads). Inertias on both sides of the rod were selected to have a 2:1 load-motor inertia ratio as is common in industry, and a resonant frequency of about 100 Hz. The compliance of the rod was calculated as 372 Nm/rad. Given (2), $J_M = 0.0014 \text{ kg-m}^2$ and $J_L = 0.0028 \text{ kg-m}^2$. A block diagram is shown in Fig. 13 and a photograph is shown in Fig. 14.
inertial load of 0.000252 kg-m$^2$ was added directly to the motor shaft. A resonance of 900 Hz still had to be dealt with in addition to the primary resonance of 100 Hz under study. Such a condition is not uncommon in industrial applications, such as when a coupling has significant inertia so that a high frequency resonance exists between the motor and coupling in addition to the lower-frequency resonance between motor and load.

Finally, $b_s$, the cross-coupled viscous damping was measured. The method used was to mount the inertial wheels and, with the drive disabled, excite the resonance using a hammer and monitor the velocity response. Using the observed time constant ($\tau \sim 0.256$ sec) and (1), $b_s$ was calculated as approximately 0.008 Nm-sec/rad.

In addition, a simulation program was built based on the same topology as the test system. The simulation was written in ModelQ, a time-domain simulation program that provided both time- and frequency-domain analysis.

**Evaluation**

Seven methods were evaluated. The first phase of the evaluation was to determine if a method made significant improvement to system stability. Those methods that did were evaluated for improvement of command response and dynamic stiffness. Also, the load inertias were moved about 15 cm (20%) to change the resonant frequency by about 10%, according to (2) to see if this change would cause instability.

**Baseline system**

The baseline system used low-pass filtering only to remove the 900 Hz resonance from between the motor and the added motor-side inertia. This required a 500 Hz filter ($\zeta = 1.0$). The velocity proportional gain ($K_{VP}$) was set to 0.42 A-sec/rad and $K_{VI}$ to 60 rad/sec; these are aggressive gains. The step response of the physical system is shown in Fig. 15. All other systems will be tuned with similarly aggressive gains to allow side-by-side comparisons of the different methods.

![Fig. 15. Step motor velocity response of the resonant load system with low pass filtering](image)

The model, which includes tools for frequency domain analysis, shows this system to have a bandwidth of about 22 Hz. The dynamic stiffness, $T_DV_E$, was evaluated at 2 Hz as 0.22 Nm-sec/rad.

**Low-pass filter**

The low-pass filter was ineffective at helping the 100 Hz resonance. This is because the filter bandwidth had to be reduced so far below the 100 Hz resonant frequency to be effective that it destabilized the loop forcing servo gains lower. The low-pass filter was not evaluated further.

**Notch filter**

The notch filter was added to the control loop. The center frequency was set experimentally to 94 Hz with the damping ratio set to 0.4. The 500 Hz low-pass filter remained to improve the 900 Hz resonance. The notch filter allowed $K_{VP}$ to be raised to 0.7 A-sec/rad ($K_{VI}$ remained at 60 rad/sec) providing 32 Hz bandwidth and dynamic stiffness at 2 Hz of 0.4 Nm-sec/rad. The step response for the notch filter is shown in Fig. 16.

![Fig. 16. Step motor velocity response of the resonant load system with a notch filter](image)

**Bi-quad filter**

Next the bi-quad filter was configured using the following settings determined experimentally:

- $F_N = 94$ Hz, $\zeta_N = 0.006$
- $F_D = 66$ Hz, $\zeta_D = 0.0037$

These values were best found at removing the impact of the resonating (bi-quad) term in (1). $K_{VP}$ was raised to 1.0, allowing the highest bandwidth of any of the methods (47 Hz) and high dynamic stiffness at 2 Hz (0.47 Nm-sec/rad).

As expected the bi-quad method was the most sensitive to the placement of the load inertia wheels. When they were moved 15 cm closer to the motor, the system became unstable. This was the only method to demonstrate this behavior. The bi-quad also controlled the load poorly, as expected. While there was no feedback device placed on the load, it could be observed that the load oscillated more with the bi-quad than with any other method, although the stability of the motor was approximately equivalent to the other methods. The motor step response of the bi-quad is shown in Fig. 17.

**Acceleration feedback**

Acceleration feedback was implemented using observed acceleration from the rigid-body observer. The acceleration signal from the compliant-body observer was not as effective as from the rigid body. Average acceleration (double differentiating position) was also less effective. The rigid-body observer was configured with the Eigenvalues of 200 Hz, well above the 100 Hz resonance. Using nominal values for motor inertia ($J_M = J_M$, Fig. 9) supported an acceleration feedback gain of $K_A = 2.0$. Acceleration feedback, because of its high-
frequency gain, excited the 900 Hz resonance more and so required the low-pass filter to be lowered to 200 Hz. Even so, KVP could be raised to 0.85 allowing much higher stiffness (0.45 Nm-sec/rad at 2 Hz) and slightly higher bandwidth than the baseline system (25 Hz vs. 22 Hz).

**Fig. 17.** Step motor velocity response of the resonant load system with bi-quad filtering

It was determined by experimentation that lowering the observer inertia from the nominal value of 0.001875 to 0.00085 kg-m\(^2\) allows substantial improvement. Here KA could be raised to 2.5 and KVP could be raised to 1.2. This system had the highest stiffness (0.63 Nm-sec/rad at 2 Hz, about three times stiffer than the baseline) and a bandwidth of 37 Hz which was second only to the bi-quad filter method. The observer-based method was less sensitive to variation in the load than the bi-quad; when the load was moved 15 cm, the system remained stable.

**Fig. 18.** Step motor velocity response of the resonant load system with acceleration feedback and \( J^M = 0.001875 \) kg-m\(^2\)

**Observer filtering**

Observer filtering did not work well for this problem, probably because the resonant frequency was too low for the method. Observer filtering can help cure resonance when the Eigen values of the observer are placed well below the resonant frequency. For this problem, such placement was impractical.

Observer filtering works by relying on the commanded current to estimate the position at frequencies above the observer Eigenvalues. This aids command response and loop stability, but it does not aid dynamic stiffness. When the Eigenvalues were placed low enough for observer filtering to help resonance (about 25 - 60 Hz), the stiffness of the system in those frequencies suffered greatly. Limited experimentation with observer filtering for high-frequency resonance indicates that the method may be promising in that case. However, it was not evaluated further for the 100 Hz resonance.

**Active resonance damping**

Active resonance damping is known to work well when two sensors are used, one for the motor and the other on the load. The experiments here investigated whether active resonance damping could be used based on the observed motor and load speed from the compliant observer. The test results showed little improvement of resonance properties when using active resonance damping. It is felt that the exceedingly low damping ratio of the physical system may have played a significant roll in the experiments. The sensitivity to this parameter is known to be extreme when the damping is very low.

**Center-of-mass control**

Center-of-mass control, like active resonance damping, is well known to work with physical sensors. As discussed above, it allows oscillatory motion on the load, but can cure loop instability because the center-of-mass does not produce net movement in the presence of resonance. In the experiments run for this paper in both the model and the physical system, center-of-mass control did allow higher values of KVP, but the motion was oscillatory and the results were judged unacceptable for this problem.

**Dynamic Stiffness**

Dynamic stiffness of the different methods was evaluated using the model with gains determined on the actual system. The low-frequency dynamic stiffness is determined by the product of KVI and KVP [5,6,10]. Fig. 19 shows that the bi-quad provided more improvement than the notch; as expected the improvement was in proportion to KVP (KVI was constant for all cases).

**Fig. 19.** Dynamic stiffness comparison for filtering methods

Dynamic stiffness for acceleration feedback is shown in Fig. 20. It has the greatest improvement over the baseline because this method allowed the highest value of KVP. Acceleration feedback also was the best method at maintaining high stiffness near the resonant frequency.
Fig. 20. Dynamic stiffness of the resonant system when using acceleration feedback

VI. CONCLUSIONS

This paper evaluated seven methods of dealing with a lightly-damped 100 Hz resonance relying on the motor encoder as the sole motion feedback sensor. The test system had two resonant frequencies, one at 100 Hz and one at about 900 Hz. A low-pass filter was employed to cure the 900 Hz resonance for all methods including the baseline.

Three methods were seen to be effective on the test system: the notch filter, the bi-quad filter, and rigid-body observer-based acceleration feedback. Each raised the system bandwidth and dynamic stiffness above the baseline system. The bi-quad filter was the most effective at raising bandwidth; acceleration feedback was most effective at increasing dynamic stiffness. Table 1 summarizes the results. Note that “Acc fb #1” uses the nominal value for the model inertia ($J^M$) while “Acc fb #2” uses about half the nominal value.

The bi-quad filter was the most sensitive to load changes. It was the only method that became unstable when the load inertias were moved 15 cm towards the motor. It also produced the most oscillatory motion on the load.

Four of the methods (low-pass filter, observer filtering, active resonance damping, and center-of-mass control) did not work well on the test system. Low-pass filtering will and observer filtering should work on higher frequency resonances. Active damping and center-of-mass control work with physical sensors but implementation using the compliant observer did not prove to be effective for this test system.

VII. ACKNOWLEDGMENTS

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Table 1. Comparison of anti-resonance control methods

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VIII. REFERENCES